Name: Francis **Math 8H 2025 Lesson 1 Basic Operations** Date ____________

1. Indicate whether if the following statements are TRUE or FALSE:

$a-b=a+(-b)$ True	$-a+b=b-a$ True	$a-(-b)=a+b$ True
$-a-(-b)=b+a$ False	$b-a=-(a-b)$ True	$a+b-c+(-d)=a+b-c-d$ True

2. Evaluate each of the following by adding or subtracting

a) $1+(-2)-3+4-(-5)$ $=-1-3+4+5$ $=-4+9$ $=5$	b) $12-(-15)+(-20)$ $=12+15-20$ $=27-20$ $=7$	c) $8+(-12)-(-13)$ $=8-12+13$ $=-4+13$ $=9$
d) $8+(-9)-(-11)-12$ $=8-9+11-12$ $=8+11-9-12$ $=19-21$ $=-2$	e) $-7+6-13+(-14)-(-23)$ $=6+23-7-13-14$ $=29-34$ $=-5$	f) $-20-(-15)+19-23$ $=15+19-23-20$ $=34-43$ $=-9$
g) $12+22-43+41-(-15)$ $=34-43+41+15$ $=-9+56$ $=47$	h) $-12+(-13)-14+(-15)-(-16)$ $=-12-13-14-15+16$ $=-54+16$ $=-38$	i) $-15+(-17)-19+23-(-22)$ $=-15-17-19+23+22$ $=-51+45$ $=-6$

3. Evaluate each of the following the order of operations

a) $8+2\times5$ $=8+10$ $=18$	b) $9+3\times4\div2-3$ $=9+12\div2-3$ $=9+6-3$ $=15-3$ $=12$	c) $2+3\times4-6\div2$ $=2+12-3$ $=14-3$ $=11$
d) $4+3\times5-6\div2$ $=4+15-3$ $=19-3$ $=16$	e) $8\times(-4)+12\div(6)$ $=-32+2$ $=-30$	f) $1200\div2\times10\div5\div3$ $=600\times10\div5\div3$ $=6000\div5\times3$ $=1200\div3$ $=400$

g) $48\times3\div24\div3+2$ $=144\div24\div3+2$ $=6\div3+2$ $=2+2$ $=4$	h) $3\times4+6\times5\div10+11$ $=12+30\div10+11$ $=12+3+11$ $=15+11$ $=26$	i) $12\times10\div5-2\times8$ $=120\div5-16$ $=24-16$ $=8$

4. Add the following without multiplying each term:

a) $7(x)+9(x)-11(x)$ $=16(x)-11(x)$ $=5(x)$	b) $7(13)+9(13)-11(13)$ $=16(13)-11(13)$ $=5(13)$	c) $6(7)-12(7)+11(7)+14$ $=-6(7)+11(7)+2(7)$ $=5(7)+2(7)$ $=7(7)$
d) $19(12)-17(12)+2(23)$ $=2(12)+2(23)$ Is it suppose to be 24? If not, $=114(2)-102(2)+23(2)$ $=12(2)+23(2)$ $=25(2)$	e) $6\times15+7\times(15)+9\times5$ $=13(15)+3(15)$ $=16(15)$	f) $7(13)+2(26)-3(52)$ $=7(13)+4(13)-12(13)$ $=11(13)-12(13)$ $=-1(13)$

5. Find the average for each of the following set of values

a) $4a$, $4b$, $4c$, and $4d$ $=\frac{4a+4b+4c+4d}{4}$ $=a+b+c+d$	b) 45, 54, 81, 18, 27, and 90 $(45+54+81+18+27+90)\div6$ $=(99+99+117)\div6$ $=(315)\div6$ $=52.5$	c) $72, 72, \ldots 72$ (13 *times*) and $48, 48, \ldots 48$ (11 *times*) $(72\times13+48\times11)\div24$ $=(936+528)\div24$ $=(1464)\div24$ $=61$

6. Given that $(x+y)^2 = x^2+2xy+y^2$, what is the value of $1000^2+2(1000)(-999)+(-999)^2$

$1000^2+2(1000)(-999)+(-999)^2$

$=(1000-999)^2$

$=(1)^2$

$=1$

7. Given that $(x-y)^2 = x^2 - 2xy + y^2$, what is the value of $997^2 - 2(997)(995) + (995)^2$

$997^2 - 2(997)(995) + (995)^2$

$= (997 - 995)^2$

$= (2)^2$

$= 4$

8. Dave wrote 12 tests and got an average of 68%. If he gets 85% on every new test that he writes, how many more tests will he need to until he gets an average greater than 75%?

$\frac{12 \times 68 + 85x}{12 + x} = 75$ $\quad 816 + 85x = 75(12 + x)$ $\quad 85x - 75x = 900 - 816$

$816 + 85x = 900 + 75x$ $\quad 10x = 84$

$x = 8\frac{4}{10}$

He will need to write 9 more tests.

9. Mt. Everest, the highest elevation in Asia, is 29,028 feet above sea level. The Dead Sea, the lowest elevation, is 1,312 feet below sea level. What is the difference between these two elevations?

$29028 - (-1312)$

$= 29028 + 1312$

$= 30340$ The difference between these two elevations is 30 340 feet.

10. The sum of 8 consecutive numbers is 188, what are the numbers?

$188 \div 8$

$= 23.5$

The numbers are 20, 21, 22, 23, 24, 25, 26, 27.

11. There are 12 sedans and 18 minivans. Each sedan has 5 females and each minivan has 7 males. What is the difference in the number of males and females?

$18 \times 7 - 12 \times 5$

$= 126 - 60$

$= 66$

The difference in the number of males and females is 66.

12. Each burger at a fastfood chain costs \$4.50 and each hotdog costs \$2.25. If Dave spent \$100 only on hotdogs and burgers, and has twice as many hotdogs than burgers, how many of each did he purchase?

$4.5 + 2 \times 2.25$ $\quad 100 \div 9$

$= 4.5 + 4.5$ $\quad = 11.\overline{1}$

$= 9$

Dave purchased 11 burgers and 22 hotdogs.

13. There are 90 students in a class and there are twice as many females and than males. The average score of the females in the class is 85% and the average score of the males is 70%. What is the average score of all the students in the class?

2 : 1
females males

$(85\times2+70)\div3$ $=80$

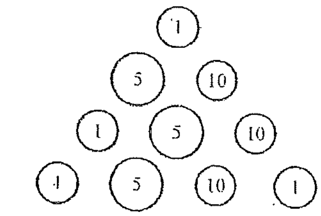
$=(170+70)\div3$

$=(240)\div3$

The average score of all the students in the class is 80%.

91

14. Micah placed pennies, nickels, and dimes in rows according to the diagram so that each row contains one more coin than the previous. What is the number of cents in the value of all the coins in the first 13 rows?



Number of coins: $1+2+3+4+5+6+7+8+9+10+11+12+13$

$=91$

Sets of 1,5,10: $91\div3=30r1$

$(1+5+10)\times30+1$

$=(16)\times30+1$

$=480+1$

$=481$

The value of all the coins in the first 13 rows is 481 cents.

Name: Francis Date:________________

Math 8H 2025 Lesson 2 Perfect Squares, Cubes, and Square Roots

1. Indicate which of the following numbers are perfect squares, cubes, both, or neither. If it is a perfect square or cube, write it as a cube or square:

a) 225 Square 15^2	b) 1024 Square 32^2	c) 243 Neither	d) 196 Square 14^2	e) 400 Square 20^2
f) 128 Neither	g) -1 Cube -1^3	h) 8000 Cube 20^3	i) 289 SQUARE ~~Neither~~ 17^2	j) 125 Cube 5^3
k) 343 Cube 7^3	L) -8 Cube -2^3	M) 10,000 Square 100^2	n) 64 Square Cube 8^2 4^3	o) 189 Neither
p) 800 Neither	q) 729 Cube Square 9^3 27^2	r) 0 Square Cube 0^2 0^3	s) 625 Square 25^2	t) 1331 Cube 11^3

2. Given each of the equations below, indicate whether if it is TRUE or FALSE, explain your work:

i) $\sqrt{-9} = 3$ TRUE or FALSE

$3^2=9$

ii) $\sqrt[3]{-64} = 4$ TRUE or FALSE

$4^3=64$

ii) If the square of "A" is equal to "B", then the square root of "B" is equal to "A" : TRUE or FALSE

Ex $A=5$ $B=25$ $A^2: 5\times5=25$ $B: \sqrt{25}=5$

iii) A number can only be a perfect square or a perfect cube, but not both: TRUE or FALSE

64 is a number that is both a perfect square and a perfect cube $8^2=64$ $4^3=64$

iv) The square root of a negative number does not exist: TRUE or FALSE

Ex. $a=2$ $a^2=2^2=4$ $(-a)^2=(-2)^2=4$

v) The cube root of a negative number does not exist: TRUE or FALSE

The cube root of a negative number is always negative

vi) Perfect squares can only be positive: TRUE or FALSE

Ex. $(a)(a)=a^2$ $(-a)(-a)=a^2$

vii) Perfect cubes can only be positive: TRUE or FALSE

A perfect cube can also be negative Ex. $-2^3=-8$

viii) Suppose "a" is an integer and not a perfect square, TRUE or FALSE then a^2 must be a perfect square

Any number must be a perfect square after multiplying it by itself. Ex $3\times3=9$ $\sqrt{9}=3$

ix) If "a" is a negative number then it can never be a TRUE or FALSE perfect cube

Both positive and negative numbers can be a perfect cube Ex $2^3=8$ $-2^3=-8$ $\sqrt[3]{8}=2$ $\sqrt[3]{-8}=-2$

x) If "a" and "b" are positive integers and are NOT TRUE or FALSE perfect squares, then $a\times b$ can be a perfect square

It can be a perfect square, but not always. Ex $a=2$ $b=32$ $2\times32=64$ $\sqrt{64}=8$

xi) Suppose "a" and "b" are prime numbers, then TRUE or FALSE $a\times b$ can never be a perfect square

This statement is true unless a and b are the same

xii) Suppose "a" and "b" are not perfect squares, then TRUE or FALSE $a\times b$ can never be a perfect square

Perfect squares are not prime numbers, they have at least 1 factor (other than itself and 1). Those factors don't have to be perfect squares.

Ex. 2 and 98 are not perfect squares, but $2\times98=196$
196 is a perfect square $14^2=196$ $\sqrt{196}=14$

3. Draw a Number Line and Estimate each of the following

a) $\sqrt{50}$ ~7 ~$\sqrt{50}$ 0 5 10	b) $\sqrt{180}$ ~13 ~$\sqrt{180}$ 0 5 10 15	c) $\sqrt{77}$ ~9 ~$\sqrt{77}$ 0 5 10
d) $\sqrt{134}$ ~12 ~$\sqrt{134}$ 0 5 10 15	e) $\sqrt{200}$ ~14 ~$\sqrt{200}$ 0 5 10 15	f) $\sqrt{63,859,102}$ ~8000 (64000000) $\sqrt{63,859,102}$ 0 5000 10000
g) $\sqrt{0.0000485}$ ~0.007 $\sqrt{0.0000485}$ 0 0.005 0.01	h) $\sqrt{2385029}$ ~1544 $\sqrt{2385029}$ 0 500 1000 1500 2000	i) $\sqrt{0.0023501}$ ~0.05 $\sqrt{0.0023501}$ 0 0.05

4. Suppose "a" is a perfect square, what numbers can the units digit be?

The units digit can be : 0,1,4,5,6,9

1×1=1 2×2=4 3×3=9 4×4=16 5×5=25 6×6=36 7×7=49 8×8=64 9×9=81 0×0=0

5. Suppose "A" and "B" are single digit positive integers, which of the following can be a Perfect Square?

i) $7ABC4$ ii) $8ABC2$ iii) $9ABC6$ iv) $75ABC44$

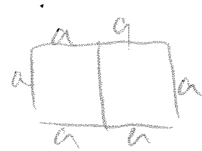
All of them except ii) 8ABC2 can be a perfect square.
It is because the ones digit of any perfect square will never be 2,3,7,or 8.

6. A square has a perimeter of 28cm. What is the area of the square in cm^2?

$(28\div4)^2$
$=(7)^2$
$=49$ The area of the square is $49\,cm^2$

7. Two squares, each with an area of 30cm^2, are placed side by side to form a rectangle. What is the perimeter of this rectangle? Give your answer to 3 decimal places:

$(\sqrt{30})8$ $\sqrt{30}\times6$



$a=\sqrt{30}$
$6a=6\sqrt{30}$
≈ 32.862

$=\sim43.818$

The perimeter of this rectangle is about 43.818cm.

8. A cube has a volume of 125cm^3. What is the area of one face of the cube?

$(\sqrt[3]{125})^2$
$=(5)^2$
$=25$

The area of one face of the cube is $25cm^2$.

9. What is the "RULE of 5's"? What is the trick to squaring a number that ends with 5? Ie: 125 x 125 = ?

$125 \times 125 = (125+5)(125-5)+25$ The trick to squaring a number that ends with 5:

$= (130)(120)+25$ x = a number that ends in 5

$= 15625$ $x^2 = (x+5)(x-5)+25$

10. Suppose "A" is a single digit positive integer, what is the value of $A5 \times A5$ in terms of "A" ?

$(5A)(5A) = 25A^2$

Ex. $A=4$ $4 \times 5 = 20$ $20 \times 20 = 400$ $4^2 \times 25 = 16 \times 25 = 400$ $= A(A+1) \times 100 + 25$

11. Square root the following without using a calculator:

a) $\sqrt{15625}$ $(120)(130)+25$ $=125$ $=15600+25$ $=15625$	b) $\sqrt{42025}$ $(210)(200)+25$ $=205$ $=42000+25$ $=42025$	c) $\sqrt{93025}$ $(310)(300)+25$ $=305$ $=93000+25$ $=93025$
d) $\sqrt{65025}$ $(250)(260)+25$ $=255$ $=65000+25$ $=65025$	e) $\sqrt{497025}$ $(710)(700)+25$ $=705$ $=497000+25$ $=497025$	f) $\sqrt{46225}$ $(220)(210)+25$ $=215$ $=46200+25$ $=46225$

12. Which is bigger? 100^2 or 50^3 Explain your answer:

$100^2 = 10000$ 50^3 is bigger than 100^2.

$50^3 = 125000$

13. In the following equations, the letters a, b and c represent different numbers.

$1^3 = 1$

$a^3 = 1+7$ $a=2$

$3^3 = 1+7+b$ $27 = 8+b$ $b=19$

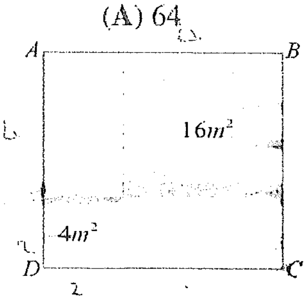
$4^3 = 1+7+c$ $64 = 8+c$ $c=56$

The numerical value of $a+b+c$ is

(A) 58 (B) 110 (C) 75 (D) 77 (E) 79

14 $ABCD$ is a square that is made up of two identical rectangles and two squares of area 4 cm^2 and 16 cm^2. What is the area, in cm^2, of the square $ABCD$?

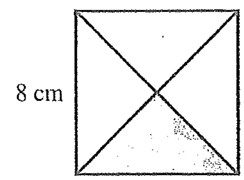
(A) 64 (B) 49 (C) 25 (D) 36 (E) 20



15. The diagonals have been drawn in the square shown. The area of the shaded region of the square is

(A) 4 cm^2 (B) 8 cm^2 (C) 16 cm^2

(D) 56 cm^2 (E) 64 cm^2

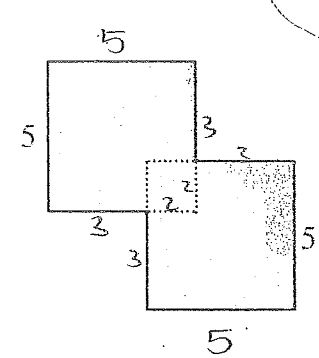


16.

Two squares, each with side length 5 cm, overlap as shown. The shape of their overlap is a square, which has an area of 4 cm^2. What is the perimeter, in centimetres, of the shaded figure?

(A) 24 (B) 32 (C) 40

(D) 42 (E) 50



17. Given that $a^2-b^2=(a+b)(a-b)$, what is the value of 1000^2-999^2 ?

1000^2-999^2

$=(1000+999)(1000-999)$

$=(1999)(1)$

$=1999$

18. If $(k+3)(k-3)=1000$, then what is the value of k^2 ?

$(k+3)(k-3)=1000$

$k^2-9=1000$

$k^2=1000+9$

$k^2=1009$

Name: Francis   Date:

Math 8H 2025 Lesson 3 Multiplication Strategies:

1. When multiplying the following numbers together, which integers should you combine together first?

2×3×4×5×7×9 =2×3×20×7×9 =120×63 =7560

I should combine 5 and 4 together first.

2. When multiplying with a two digit number like 27, how should you split it into two values to make it easier to multiply? Explain:

I should split it into integers that are easier to multiply. Numbers 2,5,10,11 are very easy to multiply. If the number isn't divisible by 2,5,10, or 11, then I should split it into two single digit numbers. Ex. 27=3×9

3. Indicate how you would use the AREA Model to multiply the following:

a) 13×7 =10×7+3×7 =70+21 =91	b) 21×8 =20×8+1×8 =160+8 =168	c) 18×9 =10×9+8×9 =90+72 =162
d) 24×7 =20×7+4×7 =140+28 =168	e) 91×8 =90×8+1×8 =720+8 =728	f) 135×8 =100×8+30×8+5×8 =800+240+40 =1080
g) 123×6 =100×6+20×6+3×6 =600+120+18 =638 =738	h) 233×7 =200×7+30×7+3×7 =1400+210+21 =1631	i) 413×8 =400×8+10×8+3×8 =3200+80+24 =3304

4. Multiply the following by breaking it down and then combining factors:

a) 25×14 =25×2×7 =50×7 =350	b) 2×5×3×7 =10×21 =210	c) 2×3×4×10 =6×40 =240	d) 24×15 =6×4×3×5 =18×20 =360	e) 35×6 =7×5×6 =30×7 =210
f) 45×8 =9×5×2×4 =10×36 =360	g) 28×12 =2×14×2×6 =84×4 =336	h) 25×7×4 =100×7 =700	i) 85×3×6 =19×5×3×2×3 =10×19×9 =10×171 =1710 1530	j) 13×11×5 =65×11 =715
k) 11×17×2 =11×34 =374	L) 123×11×3 =369×11 =4059	m) 5342×11 =58762	n) 35×12×22 =35×12×11×2 =70×12×11 =840×11 =9240	o) 77×32×25 =7×11×4×8×2.5 =100×56×11 =5600×11 =61600

5. Use the formula $(a+b)(a-b)=a^2-b^2$ to evaluate each of the following

a) 17×15 $=(16+1)(16-1)$ $=256-1$ $=255$	b) 19×21 $=(20-1)(20+1)$ $=400-1$ $=399$	c) 14×16 $=(15-1)(15+1)$ $=225-1$ $=224$	d) 81×79 $=(80+1)(80-1)$ $=6400-1$ $=6399$	e) 35×45 $=(40-5)(40+5)$ $=1600-25$ $=1575$
f) 27×33 $=(30-3)(30+3)$ $=900-9$ $=891$	g) 23×27 $=(25-2)(25+2)$ $=625-4$ $=621$	h) 45×55 $=(50-5)(50+5)$ $=2500-25$ $=2475$	i) 59×51 $=(55+4)(55-4)$ $=3025-16$ $=3009$	j) 83×77 $=(80+3)(80-3)$ $=6400-9$ $=6391$
k) 95×35 $=(65+30)(65-30)$ $=4225-900$ $=3325$	l) 35×45 $=(40-5)(40+5)$ $=1600-25$ $=1575$	m) 55×65 $=(60-5)(60+5)$ $=3600-25$ $=3575$	n) 79×71 $=(75+4)(75-4)$ $=5625-16$ $=5609$	o) 64×66 $=(65-1)(65+1)$ $=4225-1$ $=4224$
p) 72×68 $=(70+2)(70-2)$ $=4900-4$ $=4896$	q) 24×16 $=(20+4)(20-4)$ $=400-16$ $=384$	r) 62×38 $=(50+12)(50-12)$ $=2500-144$ $=2356$	s) 43×77 $=(60-17)(60+17)$ $=3600-289$ $=3311$	t) 125×35 $=(80+45)(80-45)$ $=6400-2025$ $=4375$

6. Use the Rule of 9's to evaluate the following:

a) 17×9 $=17(10-1)$ $=170-17$ $=153$	b) 45×9 $=45(10-1)$ $=450-45$ $=405$	c) 13×9 $=13(10-1)$ $=130-13$ $=117$	d) 81×99 $=81(100-1)$ $=8100-81$ $=8019$	e) 35×99 $=35(100-1)$ $=3500-35$ $=3465$
f) 17×99 $=17(100-1)$ $=1700-17$ $=1683$	g) 44×99 $=44(100-1)$ $=4400-44$ $=4356$	h) 57×999 $=57(1000-1)$ $=57000-57$ $=56943$	i) 123×999 $=123(1000-1)$ $=123000-123$ $=122877$	j) 123×9999 $=123(10000-1)$ $=1230000-123$ $=1229877$

7. Use algebra to evaluate each of the following:

a) $12(2)+22(2)-37(2)$ $=34(2)-37(2)$ $=-4(2)$ $=-8$ $-3(2)$ $=-6$	b) $13(15)-12(15)+14(15)$ $=(15)+14(15)$ $=15(15)$ $=225$	c) $15(18)-9(18)+3(18)$ $=6(18)+3(18)$ $=9(18)$ $=162$
d) $4(8)-3(24)+7(40)$ $=4(8)-9(8)+35(8)$ $=-5(8)+35(8)$ $=30(8)$ $=240$	e) $3(15)-8(20)+4(25)$ $=9(5)-32(5)+20(5)$ $=-23(5)+20(5)$ $=-3(5)$ $=-15$	f) $(17^2)+2(17)+1$ $=(17+1)^2$ $=(18)^2$ $=324$ $(3^2)^3$

9^3

8. Use the equation: $a^2-b^2=(a+b)(a-b)$ to evaluate each of the following:

a) 1000^2-999^2 $=(1000+999)(1000-999)$ $=(1999)(1)$ $=1999$	b) 501^2-499^2 $=(501+499)(501-499)$ $=(1000)(2)$ $=2000$	c) 43^2-7^2 $=(43+7)(43-7)$ $=(50)(36)$ $=1800$	d) 899^2-101^2 $=(899+101)(899-101)$ $=(1000)(798)$ $=798000$
e) 55^2-45^2 $=(55+45)(55-45)$ $=(100)(10)$ $=1000$	f) 355^2-145^2 $=(355+145)(355-145)$ $=(500)(210)$ $=105000$	g) 217^2-17^2 $=(217+17)(217-17)$ $=(234)(200)$ $=46800$	h) $(2^3)^2-4^3$ $=4^3-4^3$ $=0$

9. Use the formula $a^2 = (a+b)(a-b)+b^2$ to calculate each of the squares:

a) 99^2 $=(99+1)(99-1)+1$ $=(100)(98)+1$ $=9800+1$ $=9801$	b) 98^2 $=(98+2)(98-2)+4$ $=(100)(96)+4$ $=9600+4$ $=9604$	c) 101^2 $=(101+1)(101-1)+1$ $=(102)(100)+1$ $=10200+1$ $=10201$	d) 102^2 $=(102+2)(102-2)+4$ $=(104)(100)+4$ $=10400+4$ $=10404$	e) 51^2 $=(51+1)(51-1)+1$ $=(52)(50)+1$ $=2600+1$ $=2601$
f) 26^2 $=(26+6)(26-6)+36$ $=(32)(20)+36$ $=640+36$ $=676$	g) 44×99 $=44\times100-44\times1$ $=4400-44$ $=4356$	h) 93×93 $=(93+7)(93-7)+49$ $=(100)(86)+49$ $=8600+49$ $=8649$	i) 73×73 $=(73+3)(73-3)+9$ $=(76)(70)+9$ $=5320+9$ $=5329$	j) 81^2 $=(81+1)(81-1)+1$ $=(82)(80)+1$ $=6560+1$ $=6561$

10. For each statement, describe a situation in which the statement is true.

a. The product of two integers equals one of the integers.

$1\times3=3$

3 is equal to 3.

b. The product of two integers equals the opposite of one of the integers.

-1×7
$=-7$

Negative 7 is the opposite of positive 7.

c. The product of two integers is less than both integers.

-2×3
$=-6$

Negative 6 is less than negative 2 and positive 3.

d. The product of two integers is greater than both integers.

3×4
$=12$

12 is greater than 3 and 4.

11. If $a\times23\times b=6210$ and $a+b=N$, what is the smallest possible value of N?

$a\times b=6210\div23$ $\quad a=3\ b=9$ $\quad a=6\ b=4.5$ $\quad a=5.4\ b=5$

$a\times b=270$ $\quad 3+9=12$ $\quad 6+4.5=10.5$ $\quad 5.4+5=10.4$

If a & b must be integers, then the smallest possible value will be 12, otherwise, it is 10.4.

12. One day a sales person talked to 16 customers in 1 hour. How long would he need to work if he wanted to talk to 112 customers?

$112\div16=7$

He would need to work 7 hours if he wanted to talk to 112 customers.

13. Gaston withdrew \$26 from his bank account each week for 17 weeks. Use integers to find the total amount Gaston withdrew over the 17 weeks. Show your work.

$26\times17=442$

Gaston withdrew \$442 over the 17 weeks

14. Since sunset 6 h ago, the temperature in Brandon, Manitoba, has decreased from +1 °C to -11 °C. Predict wha temperature will be 3 h from now. What assumptions did you make?

$1-(-11)=12$ $-11-6$
$12 \div 6$ $=-17$
$=2$ 3 hours from now, the temperature will be -17°C.
2×3
$=6$ I assume that the temperature will drop in the same rate.

15. The only possible values of x are 3, 6, 9, and 12. The only possible values of y are -10, -8, -6, and -4. What is the largest and smallest value of $x \times y$?

The smallest value of $x \times y$ is -120. $12 \times -10 = -120$
The largest value of $x \times y$ is -12. $3 \times -4 = -12$

16. The mean daily high temperature in Rankin Inlet, Nunavut, during one week in January was -20 °C. What might the temperatures have been on each day of the week? How many different possible answers can you find? Explain.

Theoretically, there are infinite possible answers. Here are some of the possible answers:
1. Monday: -21°C Tuesday: -17°C Wednesday: -19°C Thursday: -23°C Friday: -20°C Saturday: -21°C Sunday: -1
2. Monday: -19°C Tuesday: -15°C Wednesday: -21°C Thursday: -25°C Friday: -24°C Saturday: -20°C Sunday: -16°C

17. The mean of a group of six numbers is 40. The number 12 is removed from the group, what is the new mean?

$\frac{40 \times 6 - 12}{5}$
$= \frac{240-12}{5}$
$= \frac{228}{5}$ $= 45\frac{3}{5}$ or 45.6 The new mean is $45.6 / 45\frac{3}{5}$.

18. In the computation shown, X, Y, Z represent a different digit respectively. Determine the value of X.

$$\begin{array}{r} X\,Y \\ \times \quad Z\,6 \\ \hline 3\,1\,2 \\ 3\,1\,2 \\ \hline 3\,4\,3\,2 \end{array}$$

$312 \div 6 = 52$
$x = 5$

19. The prime number 1999 can be written as $a^2 - b^2$. Given that $a^2 - b^2 = (a+b)(a-b)$, what is the value of $a^2 + b^2$?

$1999 = 1 \times 1999$

$a+b=1999$ $a+b=1999$
$+$ $a-b=1$ $-$ $a-b=1$
$2a=2000.$ $2b=1998$
$a=1000$ $b=999$

$a^2+b^2=1000^2+999^2$

20. The product of 119 integers is negative. At most, how many of those numbers must be negative? Explain your answer in words.

At most, all of the 119 integers must be negative. It is because when the amount of negative integers is odd, then the product must be negative. When the amount of negative integers is even, then the product must be even.

21. There are four positive numbers, a, b, c, and d (not necessarily integers). Obviously there are six ways to multiply pairs of them: ab, ac, ad, bc, bd, and cd. If I tell you that five of the six pairs of products are 2, 3, 4, 5, and 6, what is the product of the last pair.

$ab \times cd = abcd \qquad 2 \times 6 = 12$

$ac \times bd = abcd \qquad 3 \times 4 = 12$

$ad \times bc = abcd \qquad 5 \times y = 12$

$y = \frac{12}{5}$

$y = 2.4$

The product of the last pair is 2.4.

22. Challenge: A farmer grows 3000 bananas, and wants to take them to the market to sell. The market is 1000 miles away and the only way he can get there is by a hungry camel that can carry a maximum of 1000 bananas at a time. In addition, the camel needs to eat one banana to refuel for every mile that he walks. What is the maximum number of bananas that the farmer can successfully get all the way to the market?

b=3d

1002b

998

1332

1032

434

500

1002 b

1000

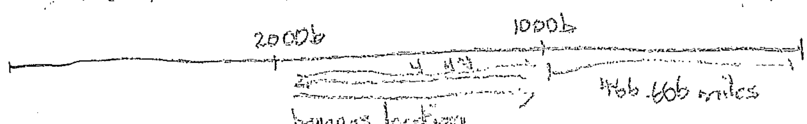
− 444

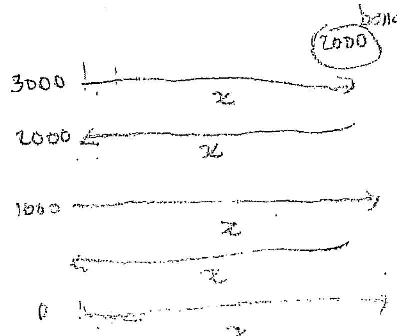
1444

445

1 333km − 334b

2 333km − 667b





$5x = 1000 \quad x = 200$ first drop off

$3y = 1000 \quad y = \frac{1000}{3} \quad y = 333.\overline{3}$

$200 + 333.\overline{3} = 533.\overline{3}$ miles walked

$1000 - 466.6 = 533.\overline{3}$ bananas

5 times of carrying

to leave 2000 bananas at x point

bc maximum carrying is 1000

Name: Francis　　　　　　　　　　　　　　Date: ____________________

M8H 2025 Lesson 4 Order of Operations:

1. Evaluate each of the following operations. Remember the order of the operations. Show all your steps:

a) $2+5\times4$ $=2+20$ $=22$	b) $-5-8\div2$ $=-5-4$ $=-9$	c) $8\times2+6$ $=16+6$ $=22$
d) $-9\times7-20$ $=-63-20$ $=-83$	e) $8\div2+6$ $=4+6$ $=10$	f) $12-6\times(-3)$ $=12+18$ $=30$
g) $3+11\times4-21$ $=3+44-21$ $=47-21$ $=26$	h) $-6+24\div8-2$ $=-6+3-2$ $=-3-2$ $=-5$	i) $-12-18\div9-6$ $=-12-2-6$ $=-14-6$ $=-20$
j) $(7+2)\times4-5+12\div2$ $=(9)\times4-5+6$ $=36+1$ $=37$	k) $11\times(3+4)-12\times2$ $=11\times(7)-24$ $=77-24$ $=53$	L) $-20\div(12+8)-15\div5+1$ $=-20\div(20)-3+1$ $=-1-2$ $=-3$
m) $2(12\div3+4)+12\div4$ $=2(4+4)+3$ $=2(8)+3$ $=16+3$ $=19$	n) $(-8\times2+12\div3)\div4\times3$ $=(-16+4)\div4\times3$ $=(-12)\div4\times3$ $=-3\times3$ $=-9$	p) $12\div4\times3\div6\times8\div4$ $=\frac{12\times3\times8}{4\times6\times4}$ $=3$
Q) $(12+2)\times4-(6\times3\div2+12)$ 14 $=(14)\times4-(18\div2+12)$ $=56-(9+12)$ $=56-21$ $=35$	r) $4+(-3-2)\times(14-2)+9\times(6-2)$ $=4+(-5)\times(12)+9\times(4)$ $=4-60+36$ $=-56+36$ $=-20$	s) $3(1-(5\times(5+2)+1)-2)$ $=3(1-(5\times(7)+1)-2)$ $=3(1-(35+1)-2)$ $=3(1-36-2)$ $=3(1-38)$ $=3(37)$ $=111$

2. Use BEDMAS to evaluate each of the following:

a) $4+5^2$ $=4+25$ $=29$	b) 3×2^4 $=3\times 16$ $=48$	c) $11+3\times 2^3$ $=11+3\times 8$ $=11+24$ $=35$
d) $3\times 2+3^3$ $=6+27$ $=33$	e) $2^2+3^2+4^2$ $=4+9+16$ $=29$	f) $4-(1+2)^2$ $=4-(3)^2$ $=4-9$ $=-5$
g) $2(3+4)^2-10$ $=2(7)^2-10$ $=2(49)-10$ $=98-10$ $=88$	h) $(4)(1+2)^2$ $=(4)(3)^2$ $=(4)(9)$ $=36$	i) $\left(\sqrt{12+4}\right)-3^2$ $=\left(\sqrt{16}\right)-9$ $=4-9$ $=-5$
j) $\sqrt{3^2+4^2}$ $=\sqrt{9+16}$ $=\sqrt{25}$ $=5$	k) $3-2^3\times 4$ $=3-8\times 4$ $=3-32$ $\cancel{=29}$ $=-29.$	L) $3^3-2^2+1^1$ $=9-4+1$ $=6$
m) $5\times 3^2-4$ $=5\times 9-4$ $=45-4$ $=41$	n) $(-2)^2+3$ $=4+3$ $=7$	p) -2^2+6 $=-4+6$ $=2$
Q) $40\div 2\times 3^2-4$ $=20\times 9-4$ $=180-4$ $=176$	r) $\dfrac{3^3-2^2+5}{12\div 3}$ $=\dfrac{9-4+5}{4}$ $=\dfrac{10}{4}$ $=2\frac{2}{4}$ $=2\frac{1}{2}$ $3^3=27$ $27-4+5$ $=\dfrac{28}{4}$ $=7$	s) $\dfrac{(21-17)\div 3}{10^2\div 20}$ $=\dfrac{(4)\div 3}{100\div 20}$ $=\dfrac{\frac{4}{3}}{5}$ $=\frac{4}{3}\div 5$ $=\frac{4}{15}$

t) $2\times(14\div2)^2+5\times12$ $=2\times(7)^2+60$ $=2\times49+60$ $=98+60$ $=158$	u) $4\times(13+8)-8^2\div(2\times4)$ $=4\times(21)-64\div(8)$ $=84-8$ $=76$	v) $(34+12)\times8\div2+2^5$ $=(46)\times4+32$ $=184+32$ $=216$
w) $36\div(6+3)\times(3^3+17)\div4$ $=36\div(9)\times(27+17)\div4$ $=4\times(44)\div4$ $=44$	x) $(3^4\div9)+32-(5\times10)+6$ $=(81\div9)+32-(50)+6$ $=(9)+38-50$ $=47-50$ $=-3$	y) $18+(57-38)\times10+4^2$ $=18+(19)\times10+16$ $=18+190+16$ $=224$

3. What operations can be placed into the boxes so that the expression will be true:

a) $12\ \boxed{+}\ 2\ \boxed{\times}\ 3\ \boxed{-}\ 1=17$	b) $9\ \boxed{-}\ 14\ \boxed{\div}\ 2\ \boxed{\times}\ 3=-12$	c) $9\ \boxed{\times}\ 4\ \boxed{-}\ 44\ \boxed{\div}\ 2=14$

4. Indicate the all mistakes in each of the following examples shown below. There are at least one mistake in each example:

a) Julie's Work: $15\div3+2-7\times4$ $=15\div5-28$ $=3-28$ $=-25$ Julie's mistake is doing the 3+2 first. The correct way $15\div3+2-7\times4$ $=5+2-28$ $=-21$	b) Tim's work: $-27+9\times2-6$ $=-27+9\times(-4)$ $=-27+(-36)$ $=63$ $-27+(-36)$ $=-63$ Tim subtracted 6 from 2 first, which is incorrect. $-27+18-6$ $=-33+18$ $=-15$ Correct way	c) Tracy's Work: $-3-4-5\times2+2\div(-1)$ $=-7-10+-2$ $=-15$ $-10+(-2)$ is equal to $-10-2$, not $-10+2$. $-7-10-2$ $=-19$

5. Where can you insert a pair of brackets into the following expression so that the value can be maximized?

$$3+6\times9+2-5\times4$$

$(3+6)\times(9+2-5)\times4$
$=(9)\times(6)\times4$
$=54\times4$
$=216$

6. Jason wrote six math exams and got the following scores: 87%, 74%, 65%, 92%, 78%, and 99%. What is the average score for his six exams?

$(87+74+65+92+78+99)\div6$
$=495\div6$
$=82.5$ The average score of his six exams is 82.5%.

7. Thomas plays on the school basketball team. He gets 3 points if he scores a 3pointer, 2 points for fields and 1 point for each foul shot made. He scored 4 3pointers, 6 fg, and 9 free throws. How many points d get?

$4\times3+6\times2+9$

$=12+12+9$

$=33$

He got 33 points.

8. A certain small factory employs 98 workers. Of these 10 receive a wage of \$200 per day and the rest receive \$100 per day. To the management, a week is equal to 6 working days. How much does the factory pay out for each week?

$(10\times200+88\times100)\times6$

$=(2000+8800)\times6$

$=(10800)\times6$

$=64800$

The factory pays out \$64800 for each week.

9. The final grade in a course is the average of the scores on 10 tests. Each test is graded on a scale of zero to 100 inclusive. A student's average on the first 7 tests was 84. The final grade of the student in the course was 63. What was the average student grade on the last 3 tests?

$(63\times10-84\times7)\div3$

$=(630-588)\div3$

$=(42)\div3$

$=14$

The average student grade on the last 3 tests is 14 out of 100.

10. Use numbers 1, 2, 3, and 4, each once to replace variables in $a+b\times c^d$. What is the maximum value of the expression? $a=1$ $b=2$ $c=3$ $d=4$

$1+2\times3^4$

$=1+2\times81$

$=1+162$

$=163$

The maximum value of this expression is 163.

11. Challenge: How many digits are in the value of $2^{2003}\times5^{1952}\div4^{27}$?

① $2\times5=1\underline{0}$

② $2^2\times5^2=1\underline{00}$

③ $2^3\times5^3=1\underline{000}$

④ $2^{1952}\times5^{1952}=1\underbrace{000\ldots}_{1952}$

⑤ $4^{27}=(2^2)^{27}$

⑥ $\Rightarrow 2^{2003}\times5^{1952}\div2^{54}$

$\dfrac{2^{2003}\cdot5^{1952}}{2^{54}}$

$=2^{1949}\cdot5^{1952}$

$=2^{1949}\cdot5^{1949}\cdot5^3$

$=10^{1949}\cdot125$

$=\underbrace{125}_{3}\underbrace{000000\ldots}_{1949}$

$1949+3=1952$

1952 digits

Name: Francis

Date:______________

Math8H 2025 Lesson 5 Divisibility Rules

1. How many of the following numbers are divisible by 3? (No calculators)

a) 115 1+1+5=7 Not divisible by 3	b) 285 2+8+5=15 Divisible by 3	c) 498 4+9+8=21 Divisible by 3	d)9381 9+3+8+1=21 Divisible by 3	e) 3951 3+9+5+1=18 Divisible by 3	f) 52376 5+2+3+7+6=23 Not divisible by 3

2. How many of the following numbers are divisible by 11? (No calculators)

a) 4013 5-3=2 Not divisible by 11	b) 4301 4-4=0 Divisible by 11	c) 30932 14-3=11 Divisible by 11	d)7392 16-5=11 Divisible by 11	e)69319 18-10=8 Not divisible by 11	f)495614 10-19=-9 Not divisible by 11

3. How many of the following numbers are divisible by 7? (No calculators)

a)1645 164-10=154 Divisible by 7	b) 4398 439-16=423 Not divisible by 7	c)23030 230-6=224 Divisible by 7	d)46231 4623-2=4621 462-2=460 Not divisible by 7	e)18557 1855-14=1841 184-2=182 Divisible by 7	f)82311 822-18=804 8231-2=8229 Not divisible by 7

4. Given that the following numbers are all divisible by 3, what are the values of "A"?

a) $4A3$ A=2 4+2+3=9 A=5 4+5+3=12 A=8 4+8+3=15	b) $3981A$ A=0 3+9+8+1+0=21 A=3 3+9+8+1+3=24 A=6 3+9+8+1+6=27 A=9 3+9+8+1+9=30	c)392AA 3+9+2=14 A=2 3+9+2+2+2=18 A=5 3+9+2+5+5=24 A=8 3+9+2+8+8=30	d)29A314A 2+9+3+1+4=19 A=1 2+9+1+3+1+4+1=21 A=4 2+9+4+3+1+4+4=27 A=7 2+9+7+3+1+4+7=33

5. Given that the following numbers are all divisible by 11, what are the values of "A"?

a) $6A2$ 6+2=8 A=8 8-(6+2)=0	b) $1234A$ 2+4=6 A=2 2+4-(1+3+2)=0	c)356A2A 3+6+2=11 A=3 11-(5+3+3)=0	d) 356AA No matter what A is, this number will not be divisible by 11

6. Indicate if the following statements are TRUE or FALSE:
   a) If a number is divisible by 9, then it must be divisible by 3
      True
   b) If a number is divisible by 3, then it must be divisible by 9
      False
   c) If a number is divisible by 2 and 4, then it must be divisible by 8
      False
   d) All even numbers that are divisible by 3 are also divisible by 6
      True
   e) If a number is divisible by 5, then the last digit must be a 0
      False
   f) The number 3AA78 can never be divisible by 11
      True
   g) If "A" is divisible by 3 and "B" is divisible by 3, then A+B is also divisible by 3
      True

7. If the 5-digit number $1732p$ is divisible by 9, determine the value of p.

1+7+3+2=13 P=5

The value of "p" is 5.

8. What digit can replace K so that the number $9K73K0$ is divisible by 6?

9+7+3=19 K=1 19+1x2=21 K=4 19+4x2=27 K=7 19+7x2=33

The digits 1,4,7 can replace K so that the number is divisible by 6.

9. Suppose the 6 digit number 2A5A93 is divisible by both 3 and 11, what are the possible values of the single digit number "A"?

Divisibility rule of 3: 2+5+9+3=19 A can be 1,4,7

Divisibility rule of 11: 2+5+9-2A-3=13-2A A can only be 1

The only possible value for "A" is 1.

10. What is the smallest positive integer that is divisible by 2, 3, 4, 5, and 6?

60

11. A boy can divide his marble collection into even groups of 3, 4, or 6. What is the smallest number of marbles in his collection?

12

12. What is the smallest 3 digit number that is divisible by the first 3 prime as well as the first 3 composite numbers?

First 3 prime: 2,3,5 First 3 composite: 4,6,8

LCM: 30 LCM: 24

The smallest 3 digit number that is divisible by these 6 numbers is 120.

13. The number $3N+63$ is divisible by 7. Explain whether N would be divisible by 7.

63 is divisible by 7 (7x9=63), so we can ignore this number. Now, it becomes "3N is divisible by 7".

Besides from the multiples of 21, the multiples of 7 cannot be divisible by 3, so N must be 7. 7x3=21

14. Use the digits 4, 5, 7, 9, and one additional digit, construct the largest possible 5-digit number divisible by 6.

9+5+7+4=25 largest possible additional digit is 8 9+5+7+4+8=33

The largest possible number divisible by 6 is 98754

15. Find the least perfect square number which is divisible by each of the numbers 8 ,12 ,15 and 20

Number: 8 12 15 20

Least perfect square: 16 36 225 100

2^3 3×2^2 3×5 $2^2 \times 5$

$N = 2^4 \times 3^2 \times 5^2$

$= 3600$

16. It is given that a number is divisible by both 6 and 26. Name two other factors of the number. Show your work.

Since it is divisible by 26, it will also be divisible by 13 and 2.

17. The integers a and b are both divisible by 2. Determine and explain whether each of the following statements would be always true or not. Provide a counter example to prove that a statement may not always be true. [Hint: If you are stuck, consider plugging in numbers for a and b and see if you can determine a trend.]

a. $a+b$ is divisible by 2 True
Since both a and b are divisible by 2, they must be even integers. The sum of 2 even integers is always even, therefore it is divisible by 2.

b. $a-b$ is divisible by 2 True
When an even number is subtracted from another even integer, the difference will be even.

c. $a+b$ is divisible by 4 False
It isn't always true.
For example, a=2 b=4
2+4=6, and 6 is not divisible by 4.

d. a^2+b^2 is divisible by 4 True
"a" and "b" can both be written out as 2 times something. Therefore a^2+b^2 is also $(2\cdot x)^2+(2\cdot y)^2$
$4x^2+4y^2$. Since a^2 and b^2 are both divisible by 4, the sum is also divisible by 4

e. ab is divisible by 4 True
"a" and "b" can both be written out as 2 × something.
Because of that, ab can be written out as
$2\cdot 2\cdot x\cdot y$, which is $4xy$. Since $4xy$ must be divisible by 4, ab must also be divisible by 4.

Challenge Section:

18. When Rachel divides her favourite number by 7, the remainder is 5. What will the remainder be if Rachel multiply her favourite number by 5 then divide by 7?

Rachel's favourite number $\rightarrow x\frac{5}{7}$ $\qquad x\frac{5}{7}\cdot 5 = x\frac{25}{7} = x{+}3\frac{4}{7}$

The remainder will be 4

19. The integers r, s, and t are three consecutive integers. Their sum is always divisible by at least 2 integers. What are those two numbers?

$x, x+1, x+2 = 3x+3$
$x-1, x, x+1 = 3x$

3 and 5

20. How many of the integers between 1400 and 2400, inclusive are an integer multiple of either 15 or 16 (or both)?

LCM=240

$1400 \div 15 = 93.\overline{3}$ $\quad 2400 \div 15 = 160$ $\quad 160 - 94 + 1 = 67$

$1400 \div 16 = 87.5$ $\quad 2400 \div 16 = 150$ $\quad 150 - 88 + 1 = 63$

$1400 \div 240 = \sim 5.8$ $\quad 2400 \div 240 = 10$ $\quad 10 - 6 + 1 = 5$

$67 - 5 + 63 - 5 + 5$

$= 125$ $\quad$ 125 integers

21. How many numbers between 200 and 2000 are divisible by 6 or 7 but not both?

$200 \div 6 = 33.\overline{3}$ $\quad 2000 \div 6 = 333.\overline{3}$ $\quad 333 - 34 + 1 = 300$

$200 \div 7 = 28.\overline{571428}$ $\quad 2000 \div 7 = 285.\overline{714285}$ $\quad 285 - 29 + 1 = 257$

$200 \div 42 = \sim 4.76$ $\quad 2000 \div 42 = \sim 47.62$ $\quad 47 - 5 + 1 = 43$

$300 - 43 + 257 - 43$

$= 257 + 214$

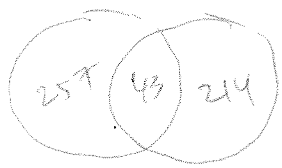
$= 471$

471 numbers

200 $\quad 34(6) \rightarrow 333(6) : 300$

$29(7) \rightarrow 285(7) : 257$

$5(42) \rightarrow 47(42) : 43$



$N = 471$

22. Ultimate Challenge: The digits 1, 2, 3, 4, and 5 are each used once to compose a five digit number *abcde* such that the three digit number *abc* is divisible by 4, *bcd* is divisible by 5, and *cde* is divisible by 3. Find the digit "a"

abc = divisible by 4 $\quad$ 6 9 12

bcd = divisible by 5 $\quad d=5$

c5e = divisible by 3 $\quad c+e=7 \quad c=4 \quad e=3$

$a=2/4 \quad b=2/4 \quad a=1 \quad b=2$

$a=1 \quad b=2 \quad c=4 \quad d=5 \quad e=3$ $\quad$ abcde = 12453 $\quad$ abc = 124 $\quad$ bcd = 245 $\quad$ cde = 453

$a=1$

Name: Francis Date:

Math 8 Honours HW Lesson 6 Prime, Factors, LCM, and GCF

1. What does it mean that two values are relatively prime? Explain?
It means the only common factor between 2 numbers is 1.

2. If the GCF of "a" and "b" is one of the values "a" or "b", then what is the relationship between "a" and 'b'?
One of the number is a multiple of the other number

3. IF the LCM of "a" and "b" is one of the values "a", or "b", then what is the relationship between "a" and 'b'?
If the LCM is "a", then the GCF will be "b". If the LCM is "b", then the GCF is "a".

4. Circle all the prime numbers below. If it's not a prime number, give one of its factors other than 1:

23	29	39 13	43	49 7	61
71	93 31	79	101	109	113
117 39	119 7	137	147 49	157	169 13

5. Given each set of numbers below, find the greatest common factor (GCF) and lowest common multiple (LCM):

a) $\langle 15,24\rangle$ $3 \mid 15\ 24$ → $5\ 8$ $\frac{15\times24}{3} = \frac{360}{3} = 120$ GCF: 3 LCM: 120	b) $\langle 18,12\rangle$ $3 \mid 18\ 12$ → $2 \mid 6\ 4$ → $3\ 2$ $\frac{18\times12}{6} = \frac{216}{6} = 36$ z GCF: 6 LCM: 36	c) $\langle 16,8\rangle$ $2 \mid 16\ 8$ → $2 \mid 8\ 4$ → $2 \mid 4\ 2$ → $2\ 1$ $\frac{16\times8}{2} = \frac{16}{1} = 16$ GCF: 8 LCM: 16
d) $\langle 35,14\rangle$ $7 \mid 35\ 14$ → $5\ 2$ $\frac{35\times14^{2}}{7^{1}} = 35\times2 = 70$ GCF: 7 LCM: 70	e) $\langle 65,91\rangle$ $13 \mid 65\ 91$ → $5\ 7$ $\frac{\overset{5}{65}\times91}{13_{1}} = 91\times5 = 455$ GCF: 13 LCM: 455	f) $\langle 195,221\rangle$ $13 \mid 195\ 221$ → $15\ 17$ $\frac{\overset{15}{195}\times221}{13_{1}} = 15\times221 = 3315$ GCF: 13 LCM: 3315

6. How many prime numbers are there less than 100? (list them out)

2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,61,67,71,73,83,89, ,97 ,59

7. Is "1" a prime number? Explain:

1 is not a prime number because it only has 1 factor. (1)

8. Check if each of the following integers are prime numbers: (Use the Prime Number TEST). If the number is NOT a prime, state of it's prime (other than 1)

a) 117 $\sqrt{117} = \sim 11$ $117 \div 3 = 39$ 3	b) 167 $\sqrt{167} = \sim 13$ $167 \div 3 =$ [illegible] $167 \div 7 = 23\frac{6}{7}$ $167 \div 11 = \sim 15$ 167 is a prime number	c) 279 $\sqrt{279} = \sim 17$ $279 \div 3 = 93$ 3
d) 913 $\sqrt{913} = \sim 30$ $913 \div 11 = 83$ → 9+3−1 = 11 11, 83	e) 493 $\sqrt{493} = \sim 22$ $493 \div 17 = 29$ I first divided by 17 because the [illegible] rules of 1–13 does not work on 493. 17, 29	f) 891 $\sqrt{891} = \sim 30$ $891 \div 3 = 297$ 3
g) 1003 $\sqrt{1003} = \sim 31$ $1003 \div 17 = 59$ 1003 is not divisible by any number from 1–13. 17, 59	h) 451 $\sqrt{451} = \sim 21$ $451 \div 17 = \sim 27$ $451 \div 19 = \sim 24$ 451 is a prime number	i) 717 $\sqrt{717} = \sim 26$ $717 \div 3 = 239$ → 7+1+7 = 15 3, 239
j) 637 $\sqrt{637} = \sim 25$ $637 \div 7 = 91$ → 63−[illegible] = 49 7	k) 217 $\sqrt{217} = \sim 15$ $217 \div 7 = 31$ → $21 - 7 \times 2 = 21 - 14 = 7$ 7, 31	L) 323 $\sqrt{323} = \sim 18$ $323 \div 17 = 19$ 17, 19

9. Suppose "A" and "B" are integers with a GCF of "x" and a product of "y". What is the LCM in terms of "x" and "y"?

$\frac{y}{x}$

10. Suppose 1316 and 2820 have a GCF of 188. What is the LCM?

$188 = 2^2 \cdot 47$

$1316 = 2^4 \cdot 47$

$2820 = 2^2 \cdot 3 \cdot 5 \cdot 47$

$LCM = 2^4 \cdot 3 \cdot 5 \cdot 47$

$2^4 \cdot 3 \cdot 5 \cdot 47$
$= 8 \cdot 3 \cdot 10 \cdot 47$
$= 24 \cdot 10 \cdot 47$
$= 1128 \cdot 10$
$= 11280$

The LCM is 11280

11. What is the smallest number with four factors?

6

6's factors: 1,2,3,6

12. Find a number between 1 to 100 that has 5 factors?

81. The factors of 81 are 1, 3, 9, 27, and 81.

13. Why do perfect squares have an "odd" number of factors? Explain:

It is because every factor comes in a pair, except the square root of the perfect square. When multiplying that factor by itself, the product will be the perfect square. Since that factor isn't in a pair, a perfect square always has an odd amount of factors. → No. factors = $x \cdot$ pairs + 1 (The square root of that perfect square)

14. What number less than 100 has the greatest number of factors?

72 : 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

15. In the multiplication shown below, each letter represents a different digit. What digit does the letter "C" represent?

$$\begin{array}{r} ABCDE \\ \times E \\ \hline EDADE \end{array}$$

A=1 E=5 D=2

$$\begin{array}{r} 1BCD5 \\ \times \quad 5 \\ \hline 501D5 \end{array} \qquad \begin{array}{r} 1BC25 \\ \times \quad 5 \\ \hline 52125 \end{array}$$

52125 ÷ 5 = 10425

$$\begin{array}{r} ABCDE \\ E \\ \hline EDADE \end{array} = \begin{array}{r} 10425 \\ \times \quad 5 \\ \hline 52125 \end{array}$$

The letter "C" represents 4.

16. Find the smallest two-digit number that is twice the product of its digits

36

3×6×2 = 18×2 = 36

17. What is the product of the LCM and GCF of two distinct numbers "a" and "b"?

12 8

LCM=24 GCF=4 24×4=96

a×b

9 15 9 × 15 = 135

LCM=45 GCF=3 3·45=135

a×b

18. Suppose that $N_1 = a^3 \times b^4 \times c^5$ and $N_2 = a^2 \times b^1 \times c^6 \times d^2$, where "a", "b", "c" and "d" are all prime factors. What is the GCF and LCM in terms of "a", "b", "c" and "d"?

The GCF is $a^2 \times b^1 \times c^5$

The LCM is $a^3 \times b^4 \times c^6 \times d^2$

19. If two numbers "a" and "b" have a GCF of 36 and a LCM of 1440, then what are the possibilities of "a" and "b" if a > b?

G.C.F : $2^2 \cdot 3^2$ 1. $a=2^5 \cdot 3^2$ $b=2^2 \cdot 3^2 \cdot 5^1$ 2. $a=2^5 \cdot 3^2 \cdot 5$ $b=2^2 \cdot 3^2$

L.C.M : $2^5 \cdot 3^2 \cdot 5^1$ 3. $a=2^5 \cdot 3^2 \cdot 5$ b= [illegible]

20. Challenge: Given that a, b, c, d, e, f, g, h, and i all represent a different digit from 1 to 9. If $\frac{ab}{cde} + \frac{fg}{hi} = 7$, then what numbers do each letter represent?

h is the smallest number

c is the second smallest

i is the third smallest

$\frac{ab}{2de} + \frac{fg}{13}$

multiples of 13 that are bigger than 200 but smaller than 300 : ~~208~~, ~~221~~, ~~234~~, 247, ~~260~~, ~~273~~, 286, 299.

No 0 repeat repeat No 0 repeat repeat

247×7=1729 98×19=1862 96×19=1824 91×19=1729 fg<91 89×19=1691 ab=38 86×19=1634 ab=95

286×7=2002 98×22=2156 2002÷22=91 fg<91

$\frac{95}{247} + \frac{86}{13} = 7$

a=9 b=5 f=8 g=6

c=2 d=4 e≈7 h=1 i=3

In the equation $\frac{ab}{cde} + \frac{fg}{hi} = 7$, a=9, b=5, c=2, d=4, e=7, f=8, g=6, h=1, i=3.

Name: Francis **Date:** ____________

Math 8H Lesson 7 Prime Factorizations (2025)

1. What is the purpose of finding the prime factorization of a number?

   After finding the prime factorization of a number, we can use it to easily find all the factors of the number and the sum of the factors.

2. When given a number "N" in the form of its prime factorization, how do you find the number of factors of "N"? ie: $N = 2^a \times 3^b \times 5^c$

   $(a+1) \times (b+1) \times (c+1)$

3. When given the number "N", with $N = 2^3 \times 6^4$, a student got the number of factors of 'n' as $4 \times 5 = 20$. What did this student do wrong? Explain:

   6 is not a prime number, that student need to turn it into prime numbers (2,3).

4. When given a number "N" in the form of its prime factorization, how do you find all the factors of "N" that are perfect squares? Ie: $N = 2^8 \times 3^9 \times 5^{10}$ .Explain:

   I will list out all the even exponents, then, I will multiply the amount of even exponents of each prime.

   $N = 2^8 \times 3^9 \times 5^{10} \longrightarrow 2^0, 2^2, 2^4, 2^6, 2^8$ (5 even exponents) $3^0, 3^2, 3^4, 3^6, 3^8$ (5 even exponents) $5^0, 5^2, 5^4, 5^6, 5^8, 5^{10}$ (6 even exponents)

   $5 \times 5 \times 6 = 150$ N has 150 factors that are perfect squares.

5. When given a number "N" in the form of its prime factorization, how do you find all the factors of "N" that are ODD numbers? Ie: $N = 2^8 \times 3^9 \times 5^{10}$ .Explain:

   ① I take all the exponents of the odd primes (includes 2^0). ② Then I start listing out the products

   $N = 2^8 \times 3^9 \times 5^{10} \rightarrow$ ① 2^0 $3^0\ 3^2\ 3^4\ 3^6\ 3^8$ $3^1\ 3^3\ 3^5\ 3^7\ 3^9$ $5^0\ 5^2\ 5^4\ 5^6\ 5^8$ $5^1\ 5^4\ 5^7\ 5^{10}$ $5^3\ 5^5\ 5^6$ ② $2^0 \cdot 3^0, 2^0 \cdot 3^1, 2^0 \cdot 3^2, 2^0 \cdot 3^3, 2^0 \cdot 3^4, 2^0 \cdot 3^5, 2^0 \cdot 3^6 \ldots$

6. When given a number "N" in the form of its prime factorization, how do you find all the factors of "N" that are EVEN numbers? Ie: $N = 2^8 \times 3^9 \times 5^{10}$ .Explain:

   ① I take all the exponents except 2^0 ② Start listing out the products

   ① $N = 2^8 \times 3^9 \times 5^{10} \rightarrow$ $2^1\ 2^4\ 2^8$ $2^2\ 2^5$ $2^3\ 2^6$ 2^7 $3^0\ 3^4\ 3^7$ $3^1\ 3^5\ 3^8$ $3^2\ 3^6$ 3^3 $5^0\ 5^3\ 5^6\ 5^9$ $5^1\ 5^4\ 5^7\ 5^{10}$ $5^2\ 5^5\ 5^8$ ② $2^1 \cdot 3^0, 2^1 \cdot 3^1, 2^1 \cdot 3^1 \cdot 5^1, 2^1 \cdot 3^2, 2^1 \cdot 3^2 \cdot 5^2 \ldots$

7. When given a number "N" in the form of its prime factorization, how do you find the SUM of all the factors of "N"? Ie: $N = 2^4 \times 3^5 \times 5^6$ .Explain:

   ① Put all the exponents of the same base in a bracket ② Add the numbers ③ Multiply the integers in the brack

   ① $(3^0+2^1+2^2+2^3+2^4)(3^0+3^1+3^2+3^3+3^4+3^5)(5^0+5^1+5^2+5^3+5^4+5^5+5^6)$

   ② $(31)(364)(19531)$ ③ $31 \times 364 \times 19531 = 220387804$

8. When given TWO or more numbers in their prime factorization form, how would you find their GCF? Explain: ie: $N_1 = 2^3 \times 3 \times 5^4$ $N_2 = 2^4 \times 3^2 \times 5^2$ $N_3 = 2 \times 3^4 \times 5^6$

   I will take the smaller exponents

   GCF of $N_1, N_2, N_3 = 2 \times 3\ 5^2$

9. When given TWO or more numbers in their prime factorization form, how would you find their LCM? Explain: ie: $N_1 = 2^3 \times 3 \times 5^4$ $N_2 = 2^4 \times 3^2 \times 5^2$ $N_3 = 2 \times 3^4 \times 5^6$

   I take the larger exponent

   LCM of $N_1, N_2, N_3 = 2^4 \times 3^4 \times 5^6$

10. Prime the Prime Factorization of each number below. Then indicate whether if it is a perfect square or perfect cube, neither, or both:

24 $=2^3\cdot 3$ Neither a perfect square nor perfect cube	800 $=2^5\cdot 5^2$ 10 80 2 5 2^4 5 Neither a perfect square nor perfect cube	864 $=2^5\times 3^3$ 2^3 108 2 54 6 9 2 3 3 3 Neither a perfect square not a perfect cube
1800 $=2^3\times 3^2\times 5^2$ 18 100 2 9 10 10 3 3 2 5 2 5 Neither a perfect square nor a perfect cube	648 $=2^3\times 3^4$ 8 81 2^3 3^4 Neither a perfect square nor a perfect cube	210 $=2\times 3\times 5\times 7$ 21 10 3 7 2 5 Neither a perfect square nor a perfect cube
5040 $=2^4\times 3^2\times 5\times 7$ 20 252 2^2 5 2^2 63 7 3^2 Neither a perfect square nor a perfect cube	3136 $=2^6\times 7^2$ 4 784 2^2 8 98 2^3 7 14 7 2 It is a perfect square	2744 $=2^3\times 7^3$ 4 686 2 343 7^3 It is a perfect cube
$N=2^2\times 50\times 5$ $=2^2\times 5^2\times 2\times 5$ $=2^3\times 5^3$ It is a perfect cube	$N=64\times 25\times 49$ $=2^6\times 5^2\times 7^2$ It is a perfect square	$N=30\times 45\times 40$ $=2\times 3\times 5\times 5\times 3^2\times 2^3\times 5$ $=2^4\times 3^3\times 5^3$ Neither a perfect square nor a perfect cube

11. Given each pair of numbers in their prime factorization, find the GCF and LCM

25 & 45 $25=5^2$ $45=3^2\times 5$ GCF $=5$ LCM $=3^2\times 5^2\rightarrow 225$	$N_1=2^2\times 3^3$ & $N_2=2^3\times 5^2$ GCF $=2^2\rightarrow 4$ LCM $=2^3\times 3^3\times 5^2\rightarrow 2700$
$N_2=2^3\times 5\times 7$ & $N_2=2\times 3^4\times 5^2$ GCF $=2\times 5\rightarrow 10$ LCM $=2^3\times 3^4\times 5^2\times 7\rightarrow 11340$	$N_1=2^3\times 4\times 6$ & $N_2=10^2\times 8$ $N_1=2^3\times 2^2\times 2\times 3$ $N_2=2^2\times 5^2\times 2^3$ $=2^6\times 3$ $=2^5\times 5^2$ GCF $=2^5\rightarrow 32$ LCM $=2^6\times 3\times 5^2\rightarrow 4800$
$N_1=a^2b^{13}c^{15}$ & $N_2=a^5b^8c^{11}d^5$ GCF $=a^2\times b^8\times c^{11}$ LCM $=a^5\times b^{13}\times c^{15}\times d^5$	$N_1=2^7$ & $N_2=3^5$ GCF $=1$ (relatively prime) LCM $=2^7\times 3^5\rightarrow 31104$

12. Use the prime factorization to find the number of factors:

$2^4\times3^2\times5^2$ $=5\times3\times3=45$	$3^4\times5^3\times11^8$ $=5\times4\times9=180$
20124 $=2^2\times3^2\times13\times43$ 4 5031 2^2 9 559 3^2 13 43 $3\times3\times2\times2$ $=36$	4500 $=2^2\times3^2\times5^3$ 45 100 3^2 5 2^2 5^2 $3\times3\times4=36$
$2^3\times3^4\times36=2^3\times3^4\times2^2\times3^2$ $=2^5\times3^6$ $6\times7=42$	$3^5\times7^{11}\times21=3^5\times7^{11}\times3\times7$ $=3^6\times7^{12}$ $7\times13=91$
$N=8^2\times3^4\times15^2$ $=2^6\times3^4\times3^2\times5^2$ $=2^6\times3^6\times5^2$ $7\times7\times3$ $=147$	$N=12^3\times20^3$ $=3^3\times2^6\times2^6\times5^3$ $=2^{12}\times3^3\times5^3$ $13\times4\times4$ $=208$

13. How do you tell if a number is a perfect square or cube by looking at the prime factorization:

If all the powers of a number's prime factors are even, then it is a perfect square.
If all the powers of a number's prime factors are divisible by 3, then it is a perfect cube.
If all the powers of a number's prime factors are divisible by 6, then it is both a perfect square and a perfect cube.

14. Find the lowest value of N such that the square root will become a positive integer:

a) $\sqrt{2^3 5^1 7^2 N}$

$N=2\times5$

$N=10$

b) $\sqrt{4^2 7^2 5^2 N}$

$N=1$

c) $\sqrt{3^4 5^3 12N}$

$=\sqrt{3^4 5^3 3^1 2^2 N}$

$=\sqrt{2^2 3^5 5^3 N}$

$N=3\times5$

$N=15$

d) $\sqrt{38412N}$

$=\sqrt{2^2 \cdot 3^2 \cdot 11 \cdot 97 \cdot N}$

$N = 97 \cdot 11$

$N = 1067$

$N = 1067$

e) $\sqrt{13992N}$

$=\sqrt{2^3 \cdot 3 \cdot 11 \cdot 53 \cdot N}$

$N = 2 \cdot 3 \cdot 11 \cdot 53$

$N = 66 \cdot 53$

$N = 3498$

$N = 3498$

f) $\sqrt{664(N-1)}$

$=\sqrt{2^3 \cdot 83 (N-1)}$

$N-1 = 2 \cdot 83$

$N = 166+1$

$N = 167$

$N = 167$

15. Find the lowest value of N such that the integer will have the indicated the indicated number of factors:

a) $2^3 3^N$ $(8\ factors)$

$=2^3 3^1$

$4 \times 2 = 8$

The lowest value of N is 1

b) $(8) \times 27N$ (48 factors)

$= 2^3 \cdot 3^3 \cdot N$

$48 = 4 \cdot 4 \cdot 3 \to N = 5^2 = 25$

$48 = 6 \cdot 8 \to N = 3^2 \cdot 2^4 = 144$

The lowest value of N is 25

c) $2^3 3^4 N^2$ $(56\ factors)$

56 is not a multiple of 5.

therefore N must be a multiple of 3

$56 = 7 \times 8$

$\to (3+1+2x)(4+1+2y)$

1. $2x = 4$ $y = 2$

~~2. $x = 3$ $y = 3$~~

1. $2^3 \times 3^4 \times (2^2 3)^2 \to 2^3 \times 3^4 \times 2^4 \times 3^2 = 2^7 \times 3^6$ $8 \times 7 = 56$

$N = 12$

The lowest value of N is 12.

16. Two positive integers have a GCF of $2 \times 3 \times 5$ and a LCM of $2^3 \times 3^4 \times 5 \times 7$. If one of the numbers is 210, find the other number.

$210 = 10 \times 21 = 2 \times 3 \times 5 \times 7$

$GCF = 2 \times 3 \times 5$

$LCM = 2^3 \times 3^4 \times 5 \times 7$

other number $= 2^3 \times 3^4 \times 5$

$= 3240$

The other number is 3240.

17. Find the smallest number N, such that $2^3 3^4 N^2$ has 56 factors.

Same as question 15c.

The lowest value of N is 12.

18. Two numbers are "*relatively prime*" if they do not share any common factors other than 1. How many positive integers less than or equal to 40 are relatively prime to 40?

1,3,7,9,11,13,17,19,21,23,27,29,31,33,37,39

16 positive integers less than or equal to 40 are relatively prime to 40.

19. Challenge: Suppose there are 1000 lockers and 1000 people. The first person opens all the lockers; the second person closes every second locker; the third person changes the state of every third locker [ie: if it's open, he closes it or if it's closed, he opens it]. This process continues, where the nth person changes the state of every nth locker. After all 1000 people have gone through, how many lockers are open?

lockers

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1st	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O
2nd		C		C		C		C		C		C		C		C		C		C
3rd			C			O			C			O			C			O		
4th				O				O				C				O				O
5th					C					O					O					C
6th						C						O						C		
7th							C							O						
8th								C								C				
9th									O									O		

Only lockers with a perfect square are open.
There are 31 perfect squares that are smaller than 1000.
Therefore 31 lockers are open.